SCITT Programme
Mathematics Training
Session Two
Calculations

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This session will include:

• Part One: **Gap Task Feedback**
• Part Two: **NCETM / Progression Maps**
• Part Three: **Mental Calculations**
• Part Four: **Properties of Number**
• Part Five: **Written Calculations**
• **Lots of maths!**
Use coloured blocks to navigate around. Check out the different resources: subject knowledge; making connections; articles; activities; exemplification; videos.

Check out the progression maps with reasoning.
PROGRESSION MAPS WITH REASONING
Part Three:
Mental Calculations
Is mental calculation the same as mental arithmetic?

For many adults, mental calculation is about doing arithmetic; it involves rapid recall of number facts – knowing your number bonds to 20 and the multiplication tables to $12 \times 12$.

Rapid recall of number facts is one aspect of mental calculation but there are others. This involves presenting children with calculations in which they have to work out the answer using known facts and not just recall it from a bank of number facts that are committed to memory. Children should understand and be able to use the relationship between the four operations and be able to construct equivalent calculations that help them to carry out such calculations.

Research shows that learning key facts ‘by heart’ enables children to concentrate on the calculation which helps them to develop calculation strategies. Using and applying strategies to work out answers helps children to acquire and so remember more facts. Many children who are not able to recall key facts often treat each calculation as a new one and have to return to first principles to work out the answer again. Once they have a secure knowledge of some key facts, and by selecting problems carefully, you can help children to appreciate that from the answer to one problem, other answers can be generated.
Six key points to remember when planning and teaching mathematics:
- every day is a mental mathematics day
- hands-on learning is important
- seeing mathematics through models and images supports learning
- talking mathematics clarifies and refines thinking
- make mathematics interesting
- learning from mistakes should build children’s confidence.

Applying these to teaching mental calculation leads to the following teaching principles:
- Commit regular time to teaching mental calculation strategies.
- Provide practice time with frequent opportunities for children to use one or more facts that they already know to work out more facts.
- Introduce practical approaches and jottings with models and images children can use to carry out calculations as they secure mental strategies.
- Engage children in discussion when they explain their methods and strategies to you and their peers.
Revisiting mental work at different times in the daily mathematics lesson, or even devoting a whole lesson to it from time to time, helps children to generate confidence in themselves and a feeling that they control calculations rather than calculations controlling them. Look out too for opportunity to introduce short periods of mental calculation in other lessons or outside lessons when queuing for some activity. Regular short practice keeps the mind fresh. Mental calculation is one of those aspects of learning where – if you don’t use it you will end up losing it!
What are the different aspects of mental calculations?

Recalling Facts

• What is 7 add 3?
• What is 6 x 9?
• How many days are there in one week? In four weeks?
• What fraction is equivalent to 0.25?
• How many minutes in an hour? In six hours?
Applying Facts

• Tell me two numbers that have a difference of 12.
• If 3 x 8 is 24, what is 6 x 0.8?
• What is 20% of £30?
• What are the factors of 42?
• What is the remainder when 31 is divided by 4?
Hypothesizing or Predicting

• Roughly, what is 51 times 47?

• On a 1 to 9 keypad, does each row, column and diagonal sum to a number that is a multiple of 3?
Designing and Comparing

- How could you subtract 37 from 82?
- How could we test a number to see if it is divisible by 6?
- How could we find 20% of a quantity?
Interpreting Results

• Double 15 then double again; now divide your answer by 4. What do you notice? Will this always work?

• If I know 5% of a length is 2cm. What other percentages can we work out quickly?
Applying Reasoning

• The seven coins in my purse total 23p. What could they be?
• In how many different ways can four children sit round a table?
• Why is the sum of two odd numbers always even?
Open and closed questions

**Closed** questions help to establish specific areas of knowledge, skills, and understanding; they often focus on children providing explanations as to how and why something works and can be applied when identifying and developing approaches and strategies for a particular purpose.

**Open** questions help to generate a variety of alternative solutions and approaches that offer children a chance to respond in different ways; they often focus on children providing explanations and reasons for their choices and decisions, and a comparison of which of the alternative answers are correct or why strategies are more efficient.

- **Open Questions Begin**
  - What
  - When
  - Who
  - Where
  - Which
  - How

- **Closed Questions Begin**
  - IS
  - DO/DID
  - CAN
  - HAS/HAVE/HAD
  - SHALL
  - WILL
PRACTISE
Part Four: Properties of Number

Basic Properties of Numbers

**Commutative**
Changing the order of addends or factors does not affect the sum or product.
- \( a + b = c \)  \( a + c = b \)
- \( b + c = a \)  \( b + a = c \)
- \( 12 + 6 = 18 \)  \( 6 + 12 = 18 \)
- \( 5 \times 7 = 35 \)  \( 7 \times 5 = 35 \)

**Associative**
The order in which numbers are grouped does not affect the sum or product.
- \( (a + b) + c = d \)  \( (a + c) + b = d \)
- \( a + (b + c) = d \)  \( a + (c + b) = d \)
- \( 3 + (5 + 2) = 10 \)  \( 3 + (5 + 2) = 10 \)
- \( 4 \times (7 \times 3) = 84 \)  \( (4 \times 7) \times 3 = 84 \)

**Distributive**
Adding two or more numbers together, then multiplying the sum by a factor is equal to multiplying each number alone by the factor first, and then adding the products.
- \( a \times (b + c) = (a \times b) + (a \times c) \)
- \( 4 \times (5 + 2) = (4 \times 5) + (4 \times 2) \)
- \( 4 \times 4 = 4 \times 32 \)
- \( 10 = 32 \)

**Identity**
The additive identity is zero. If you add zero to an addend, the sum will equal that addend.
- \( a + 0 = a \)
- \( 8 + 0 = 8 \)

The multiplicative identity is one. If you multiply a factor by one, the product will equal that factor.
- \( 0 \times 1 = 0 \)
- \( 25 \times 1 = 25 \)
It is SO important that children are taught, explicitly, the properties of number and how to use them to make their workings more efficient.
COMMUTATIVE PROPERTY

The ‘Commutative Property’ says we can swap numbers over and still get the same answer.

When we add ....

\[ a + b = b + a \]

When we multiply ...

\[ a \times b = b \times a \]
ASSOCIATIVE PROPERTY

The ‘Associative Property’ says that it doesn’t matter how we group the numbers (i.e. which we calculate first) …

When we add ….

\[(a + b) + c = a + (b + c)\]

When we multiply ….

\[(a \times b) \times c = a \times (b \times c)\]
DISTRIBUTIVE PROPERTY

The ‘Distributive Property’ is the BEST one of all, but needs careful attention.

This is what it lets us do:

\[ 3 \times (2+4) = 3 \times 2 + 3 \times 4 \]
Uses:
Sometimes it is easier to break up a difficult multiplication:

Example: What is $6 \times 204$?

$$6 \times 204 = 6 \times 200 + 6 \times 4 = 1200 + 24 = 1224$$

Or to combine:

Example: What is $16 \times 6 + 16 \times 4$?

$$16 \times 6 + 16 \times 4 = 16 \times (6 + 4) = 16 \times 10 = 160$$
We can use it in subtraction too:

Example: \(26 \times 3 - 24 \times 3\)

\[26 \times 3 - 24 \times 3 = (26 - 24) \times 3 = 2 \times 3 = 6\]

We could use it for a long list of additions, too:

Example: \(6 \times 7 + 2 \times 7 + 3 \times 7 + 5 \times 7 + 4 \times 7\)

\[6 \times 7 + 2 \times 7 + 3 \times 7 + 5 \times 7 + 4 \times 7 = (6+2+3+5+4) \times 7 = 20 \times 7 = 140\]
The Commutative Law does **not** work for subtraction or division:

Example:
- $12 / 3 = 4$, but
- $3 / 12 = \frac{1}{4}$

The Associative Law does **not** work for subtraction or division:

Example:
- $(9 - 4) - 3 = 5 - 3 = 2$, but
- $9 - (4 - 3) = 9 - 1 = 8$

The Distributive Law does **not** work for division:

Example:
- $24 / (4 + 8) = 24 / 12 = 2$, but
- $24 / 4 + 24 / 8 = 6 + 3 = 9$
IDENTITY PROPERTY

The "Multiplicative Identity" is 1, because multiplying a number by 1 leaves it unchanged:

\[ a \times 1 = 1 \times a = a \]

The "Additive Identity" is 0, because adding 0 to a number leaves it unchanged:

\[ a + 0 = 0 + a = a \]
Part Five: Written Calculations

\[ \begin{array}{c}
+ 3425 \\
\underline{+ 2718} \\
\hline
6143
\end{array} \]
Calculation Strategies

• Known fact
• Generate from a known fact
• Mental
• Mental with jottings
• Informal written method (number line/chunking)
• Formal written method
• PHONE!!! (calculator)
Parts of Subtraction

\[ 5 - 1 = 4 \]

Minuend  \( \uparrow \)  Subtrahend  \( \uparrow \)  Difference  \( \uparrow \)

Multiplication:

\[ 6 \times 3 = 18 \]

Factor (or Multiplier)  \( \uparrow \)  Factor (or Multiplicand)  \( \uparrow \)  Product  \( \uparrow \)

Parts of Addition

\[ 2 + 3 = 5 \]

Addend  \( \uparrow \)  Addend  \( \uparrow \)  Sum or Total  \( \uparrow \)
ADDITION

Not lining up the digits correctly.

Not remembering to add in any value carried over.

Not carrying over (regrouping).
SUBTRACTION

Not commutative as you can do $5 - 6$. It equals $-1$.

Start with the larger number when using the formal written method.

$0 - 6 = -6$ not $0$

Confusion can arise with the identity property.

You do not ‘borrow’ – you exchange.

Multiple exchanges eg. $200 - 145$. Work through each place. No jumping.
MULTIPLICATION

Not adding in the carry over.

Think about how it is laid out.

Use estimation to support accurate calculation.

Add a zero when completing long multiplication to hold the place.

Too many digits – caused by not crossing out.
DIVISION

All operations in use.

Need to know how to generate multiplication facts.

Use of multiplication to check.

Bus stop method!!!

Lack of understanding of place value.

Lots of opportunities for error.
WANDSWORTH CALCULATION POLICY 2014
Column subtraction

\[
\begin{array}{c}
942 - 214 \\
\text{Expanded method} \\
30 \quad 12 \\
900 \quad 40 \quad 2 \\
- \quad 200 \quad 10 \quad 4 \\
\hline
700 \quad 20 \quad 8
\end{array}
\quad \begin{array}{c}
\text{Compact Method} \\
3 \quad 12 \\
942 \\
- \quad 214 \\
\hline
728
\end{array}
\]

SIDE BY SIDE

UNITISING
REGROUPING
EXCHANGING
CARRYING

Concrete Manipulatives \quad Pictorial Representation \quad Abstract Symbols

\[
\begin{array}{c}
4 + 4 = 8 \\
2 \times 4 = 8
\end{array}
\]

Break the numbers up in tens and ones using expanded notation.
25 + 37 is the same as...

1. Add the tens
20 + 5
30 + 7
50 + 12 = 62

3. Then add the ones

3. Now add both numbers together!
## ADDITION SUBTRACTION

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Mental Calculation)</td>
<td>add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</td>
<td>add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</td>
<td>add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**FORMAL WRITTEN METHODS OF ADDITION & SUBTRACTION**

<table>
<thead>
<tr>
<th>Addition and subtraction</th>
<th>789 + 642 becomes</th>
<th>874 – 523 becomes</th>
<th>932 – 457 becomes</th>
<th>932 – 457 becomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7 8 9</td>
<td>8 7 4</td>
<td>8 12 1</td>
<td>9 3 2</td>
</tr>
<tr>
<td>+</td>
<td>6 4 2</td>
<td>– 5 2 3</td>
<td>[\underline{9 3 2}]</td>
<td>[\underline{8 5 7}]</td>
</tr>
<tr>
<td></td>
<td>[\underline{1 4 3 1}]</td>
<td>[\underline{3 5 1}]</td>
<td>[\underline{4 7 5}]</td>
<td>[\underline{4 7 5}]</td>
</tr>
<tr>
<td>Answer: 1431</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer: 351

Answer: 475

Answer: 475

Answer: 475
MULTIPLICATION

Lattice Method

<table>
<thead>
<tr>
<th>×</th>
<th>30</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>210</td>
<td>35</td>
</tr>
</tbody>
</table>

$210 + 35 = 245$

Grid Method

<table>
<thead>
<tr>
<th>×</th>
<th>30</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>30</td>
</tr>
</tbody>
</table>

$600 + 100 = 700$
$180 + 30 = 210$
$700 + 210 = 910$
FORMAL WRITTEN METHOD OF SHORT MULTIPLICATION

**Short multiplication**

- **24 \times 6** becomes
  
  \[
  \begin{array}{c}
  2 \ 4 \\
  \times \ 6 \\
  \hline
  1 \ 4 \ 4 \\
  \end{array}
  \]

  Answer: 144

- **342 \times 7** becomes
  
  \[
  \begin{array}{c}
  3 \ 4 \ 2 \\
  \times \ 7 \\
  \hline
  2 \ 3 \ 9 \ 4 \\
  \end{array}
  \]

  Answer: 2394

- **2741 \times 6** becomes
  
  \[
  \begin{array}{c}
  2 \ 7 \ 4 \ 1 \\
  \times \ 6 \\
  \hline
  1 \ 6 \ 4 \ 4 \ 6 \\
  \end{array}
  \]

  Answer: 16 446
# FORMAL WRITTEN METHOD OF LONG MULTIPLICATION

<table>
<thead>
<tr>
<th>Long multiplication</th>
<th>24 × 16 becomes</th>
<th>124 × 26 becomes</th>
<th>124 × 26 becomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 4</td>
<td>1 2 4</td>
<td>1 2 4</td>
</tr>
<tr>
<td>× 1 6</td>
<td></td>
<td>× 2 6</td>
<td>× 2 6</td>
</tr>
<tr>
<td>2 4 0</td>
<td></td>
<td>2 4 8 0</td>
<td>7 4 4</td>
</tr>
<tr>
<td>1 4 4</td>
<td></td>
<td>7 4 4</td>
<td>2 4 8 0</td>
</tr>
<tr>
<td>3 8 4</td>
<td></td>
<td>3 2 2 4</td>
<td>3 2 2 4</td>
</tr>
<tr>
<td></td>
<td>Answer: 384</td>
<td>Answer: 3224</td>
<td>Answer: 3224</td>
</tr>
</tbody>
</table>
FORMAL WRITTEN METHOD OF LONG MULTIPLICATION

\[ 327 \times 53 = 17331 \]

Remember to hold the place!
DIVISION

\[
\begin{align*}
73 \div 5 & = 14 \text{ r } 3 \\
5 & \underline{\mid 73} \\
& \quad - 50 \quad (10 \times 5) \\
& \quad \underline{23} \\
& \quad - 20 \quad (4 \times 5) \\
& \quad \underline{3} \\
& \quad 10 + 4 = 14 \\
\end{align*}
\]

How many 5s have been subtracted?
14 sets of 5, with 3 left over.

Answer: \[73 \div 5 = 14 \text{ r } 3\]

Chunking
## FORMAL WRITTEN METHOD OF SHORT DIVISION

<table>
<thead>
<tr>
<th>Short division</th>
<th>432 ÷ 5 becomes</th>
<th>496 ÷ 11 becomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>98 ÷ 7 becomes</td>
<td>432 ÷ 5 becomes</td>
<td>496 ÷ 11 becomes</td>
</tr>
<tr>
<td></td>
<td>86 remainder 2</td>
<td>45 5/11</td>
</tr>
<tr>
<td>7</td>
<td>9 8</td>
<td>45 5/11</td>
</tr>
<tr>
<td>Answer: 14</td>
<td>Answer: 86 remainder 2</td>
<td>Answer: 45 5/11</td>
</tr>
</tbody>
</table>
FORMAL WRITTEN METHOD OF LONG DIVISION

Long division

\[
\begin{align*}
432 \div 15 & \text{ becomes } 28 \text{ remainder } 12 \\
15 & \overline{)432} \\
\quad & -300 \\
\quad & 132 \\
\quad & -120 \\
\quad & 12 \\
\end{align*}
\]

Answer: 28 remainder 12

\[
\begin{align*}
432 \div 15 & \text{ becomes } 28 \frac{4}{5} \\
15 & \overline{)432} \\
\quad & -300 \\
\quad & 132 \\
\quad & -120 \\
\quad & 12 \\
\end{align*}
\]

Answer: 28 \frac{4}{5}

\[
\begin{align*}
432 \div 15 & \text{ becomes } 28.8 \\
15 & \overline{)432} \\
\quad & -300 \\
\quad & 132 \\
\quad & -120 \\
\quad & 12 \\
\end{align*}
\]

Answer: 28.8
FORMAL WRITTEN METHOD OF LONG DIVISION

Long Division
- Each person represents a step in the long division process.
- 1. Divide
- 2. Multiply
- 3. Subtract
- 4. Bring down
- 5. Repeat or Remainder

How to Divide!
- Does  ÷ (divide)
- McDonald’s x (multiply)
- Serve - (subtract)
- Cheese (compare)
- Burgers? ↓ (bring down)

20 Times Table

<table>
<thead>
<tr>
<th>20</th>
<th>x</th>
<th>=</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>60</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>80</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>100</td>
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<tr>
<td>20</td>
<td>6</td>
<td>120</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
<td>140</td>
</tr>
<tr>
<td>20</td>
<td>8</td>
<td>160</td>
</tr>
<tr>
<td>20</td>
<td>9</td>
<td>180</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>20</td>
<td>11</td>
<td>220</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
<td>240</td>
</tr>
</tbody>
</table>
# Multiplication & Division

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
<th>Year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs</td>
<td>write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Mental Methods)</td>
<td>multiply two-digit and three-digit numbers by a one-digit number using formal written layout</td>
<td>multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers</td>
<td>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</td>
<td>divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</td>
<td>use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals))</td>
</tr>
</tbody>
</table>
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